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Analysis of Atmospheric Gamma Ray Bursts Based on the Mechanism of Generation of Relativistic Electron Avalanches

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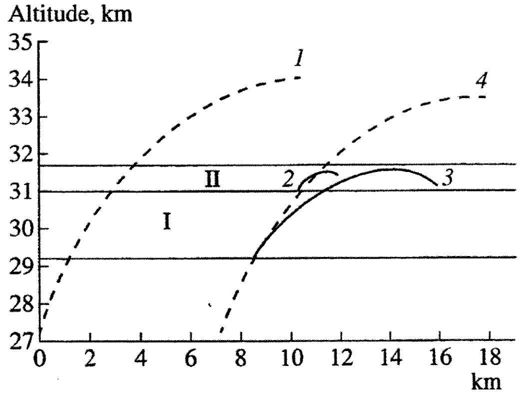
Abstract—The energy and angular distributions of high-energy electrons, obtained by simulating a relativistic runaway electron avalanche (RREA) by the Monte Carlo method, have been used (with regard to exact data on the RREA rate of development in thunderstorm electric fields) to calculate the γ ray emission of upward atmospheric discharges (UADs), taking into account the effects of the geomagnetic field. The results obtained agree with the characteristics of γ ray bursts of terrestrial origin recorded above thunderclouds on the orbital station. An agreement is an argument for the UAD model with the participation of RREA, proposed by Gurevich and Roussel-Dupré with the co-authors, and their own interpretation of γ ray bursts.

1. INTRODUCTION

A theory of UADs, based on the concept of RREA, was proposed several years ago [Gurevich *et al.*, 1992; Roussel-Dupré *et al.*, 1994, 1996]. Since RREA is capable of generating hard bremsstrahlung γ rays in the atmosphere, the development of RREA can be used to explain observations performed by Fishman *et al.* [1994], who recorded intense γ ray bursts onboard the orbital station, the sources of which were localized in the atmospheric regions of thunderstorm activity. Roussel-Dupré and Gurevich [1996] calculated the RREA γ ray emission at the satellite orbit and considered UAD as one generation of runaway electrons (REs). To make the RREA enhancement value acceptable and coordinated with UAD observations, including the γ-range, it was assumed that an avalanche starts at a certain point located much above a cloud. Specific calculations of the γ ray emission were performed for three start altitudes (25, 26, and 27 km) and for a constant overvoltage (δ) determined as the ratio of the local electric force (eE) to the minimal value of the retarding force (F_r), which characterizes the average effect of the electron energy loss [Bethe and Ashkin, 1953]. Roussel-Dupré and Gurevich [1996] discussed the effects of the geomagnetic field, which, however, were not taken into account in the calculations. The γ ray flux, calculated for $\delta = 2$, a start altitude of 27 km, and an avalanche enhancement of exp(35), agrees with the number of hard photons registered by Fishman *et al.* [1994].

The exact values of the length (l_e) and time (t_e) of the *e*-fold avalanche enhancement, obtained by the Monte Carlo (MC) and kinetic equation methods [Symbalisty *et al.*, 1997, 1998; Babich *et al.*, 1998, 2001, 2001a, 2001b], proved to be much larger than the values presented in [Gurevich *et al.*, 1992; Roussel-Dupré *et al.*, 1994, 1996], which indicates that the discharge process develops in a slightly different manner. In particular, since the rate of avalanche development is lower, an avalanche should start from much lower altitudes (factually, immediately from a thundercloud top) to become sufficiently increased. However, even in this case isolated RREA, as one RE generation, is too weak to guarantee agreement with results of observations performed by Fishman *et al.* [1994] since the horizontal geomagnetic field bends electron trajectories, as a result of which the RE flux is amplified only to a certain altitude H_0. Moreover, the lower layers of the atmosphere rather intensely absorb γ rays. To overcome these difficulties, it is necessary to take into account that not one generation of relativistic electrons participates in the development of UAD. UAD should be considered as a prolonged avalanche maintained by a constant irradiation of the atmosphere by CRs so that RREA are initiated in the entire volume between a thundercloud top and an altitude where electron trajectories bend during the existence of the electric field. The high-energy discharge stage responsible for RE generation continues when the electric field is shielded by the polarized secondary plasma generated by REs. Therefore, it is necessary to analyze the γ ray emission of UAD proceeding from lower start altitudes and new l_e and t_e. The accuracy of calculations decreases as a



Trajectories of electrons moving under the joint action of the geomagnetic field, the thundercloud electric field, and the retarding force due to the interaction with air molecules and started from the following altitudes: h = (*1* and *4*) 2, (*3*) 29, and (*2*) 31 km. I and II are the regions of RREA formation and quasi horizontal propagation of runaway electrons.

result of an incomplete knowledge of the specific features of UAD as a natural phenomenon.

The aim of this work is to corroborate the Roussel-Dupré–Gurevich hypothesis that forms the basis for an UAD model, using results of the RREA calculations obtained in [Babich *et al.*, 2001, 2001a, 2001b], by comparing the calculated characteristics of the RE γ ray emission with data of in situ measurements presented in [Fishman *et al.*, 1994]. The comparison cannot naturally be absolutely adequate because of the absence of spatial–temporal characteristics of UAD for a specific thunderstorm above which the orbital station passed during the registration of γ ray bursts of a terrestrial origin [Fishman *et al.*, 1994]. An analysis will be performed within the scope of the model including many successive RREA generations. The l_e and t_e scales of RREA are evidently not used but are implicitly included in the model via time hierarchy

$$t_e = l_e/c \ll (H_0 - H_{cl})/c \ll \Delta t_{\rm disch} \approx \Delta t_\gamma, \qquad (1)$$

where $((H_0 - H_{cl})/c$ is the duration of one RE generation in the domain between the altitudes of a cloud top (H_{cl}) and a bend of trajectories (H_0); and $\Delta t_{\rm disch}$ is the local duration of the UAD stage (controlled by REs) near H_0, which should be close to the duration of γ ray bursts (Δt_γ) in [Fishman *et al.*, 1994]. We will use the energy and angular electron distributions obtained by means of the numerical simulation of RREA.

2. EFFECT OF THE HORIZONTAL GEOMAGNETIC FIELD (LOW LATITUDES)

In [Fishman *et al.*, 1994] γ ray bursts were measured from the satellite orbiting in the equatorial plane. Therefore, we should take into account that the geomagnetic field, bending electron trajectories, could hinder the development of RREA in the equatorial regions and, consequently, the penetration of UAD from the vicinity of a cloud top to higher altitudes. For the γ ray emission of UAD, this limitation is decisive because it is necessary to take into account photon absorption in rather dense layers of the atmosphere between a charge and the satellite. The effect of the horizontal magnetic field on UAD is determined by the ecB/eE ratio, where ecB is the Lorentz force for a relativistic electron ($v_e \approx c$). For small cB/E, a charge slightly deviates from the vertical direction (direction of the electric force). The deviation, however, considerably increases as cB/E approaches unity.

As Lehtinen *et al.* [1997], we use the model of the quasistatic electric field. According to the generally accepted mechanism, the field above a thundercloud is first shielded by the polarized plasma between a cloud top and the ionosphere. As a cloud is discharged by a lightning, polarization charges near a cloud top become uncompensated, and the field equal to that of a thundercloud in the absence of shielding (according to the principle of superposition) appears above a cloud. It is assumed that the duration of a lightning, which carries away the upper charge of a cloud, is small as compared to the characteristic times of other processes. The field of polarization charges decays with increasing air conductivity as a result of UAD development.

Figure indicates the trajectories of electrons moving under the action of three forces: ecB, eE, and the retarding force (F_r). A point charge (Q_{cl} = 210 C), located at an altitude of H_{cl} = 18 km (where a cloud top was located during the observations performed by Fishman *et al.* [1994]) and, consequently, simulating the upper charge of a cloud, is the source of eE. In this paper H_{cl} = 18 km and the values of Q_{cl} are not more than 210 C, which is a large but acceptable value of a thundercloud charge. Thus, Berger [1978], Brook *et al.* [1982], and Uman [1997] rather reliably testify to the presence of charges of up to 350 C. The trajectories shown in figure have been calculated from the solution to the same equation of motion that was used in [Gurevich *et al.*, 1996; Lehtinen *et al.*, 1997]

$$d\mathbf{p}/dt = eE + \frac{e}{mc\gamma}(\mathbf{p} \times \mathbf{B}) - \nu\mathbf{p}, \qquad (2)$$

where

$$\gamma = 1/\sqrt{1-\beta^2}, \ \beta = v/c, \ \nu = F_r(p)/p$$

for the initial energy of electrons ε equal to 500 keV and for the upward initial momentum ($\mathbf{p}$). Trajectories *1* and *4* in figure correspond to electrons that started at an altitude of 27 km from the opposite ends of a charge diameter set equal to 6.5 km (see Section 4), whereas trajectories *2* and *3* correspond to electrons started at altitudes of 31 and 29 km, respectively, with approximately the same length of RREA enhancement (l_e). The estimates made based on the RREA characteristics obtained in [Babich *et al.*, 1998, 2001, 2001a, 2001b] indicated that most REs are generated precisely in this altitude range.

Figure indicates that the horizontal bending of the trajectories results in a sharp narrowing of the RE flux cross section so that REs propagate almost horizontally in a rather narrow range of altitudes. A corresponding increase in a flux density leads to a rapid relaxation of the electric field as a result of an increase in the conductivity of the air ionized by REs themselves. Since the process is nonlinear, the characteristic time of field relaxation (τ) in this region due to polarization of the plasma generated by REs is by an order of magnitude less than in the lower atmospheric layers, where the flux density of upward propagating REs and, consequently, the specific ionization rate are much less. This means that when moving in a rather narrow spatial region near an altitude of bending (H_0), REs are mostly not affected by the electric field. Everywhere in the paper, the denotation τ corresponds to the electric field relaxation due to the air conductivity generated by REs.

Although this approximate analysis has been performed for a limited range of electron energies, it indicates that, in the case of a horizontal magnetic field, a charge propagates upward to an altitude of H_0 defined as $cB \approx E$. The cB/E ratio increases with altitude as the electric field relaxes. After reaching H_0, REs move on average horizontally, i.e., perpendicularly to the electric force. Consequently, the energy accumulated by an RE flux sharply decreases and the avalanche rate decreases. Therefore, a further consideration will be based on the assumption that an avalanche abruptly stops at an altitude of H_0.

The $cB \approx E$ equality becomes valid at different altitudes, i.e., at different number densities of air molecules (N) and, consequently, at different E/N and δ, depending on a charge value and distribution in a cloud. If a considered point is located rather high above a cloud, it is not obligatory to know an actual charge distribution, so that the field of a point charge $E(H_0) = Q_{cl}/[4\pi\varepsilon_0(H_0 - H_{cl})^2]$ located at an altitude of H_{cl} is an approximation sufficient for determining the H_0 altitude, where the $E(H_0) = cB$ equality is satisfied, from the formula $H_0 = H_{cl} + [Q_{cl}/(4\pi\varepsilon_0 cB)]^{1/2}$. For $B = 40$ μT $H_{cl} = 18$ km, and $Q_{cl} \leq 210$ C, $H_0 \leq 30.5$ km. A corresponding range of local overvoltage (δ)

$$\delta(H_0) = eE(H_0)/F_{r,\min}(H_0) = ecB/[F^{(1)}_{r,\min}P(H_0)] \quad (3)$$

is not more than 4 and is not less than unity, as a critical value for RREA. Here $F^{(1)}_{r,\min} = 218$ keV m^{-1} atm^{-1} is the minimal value of the retarding force $F_r(p)$ [Bethe and Ashkin, 1953] reduced to $P = 1$ atm, corresponding to the energy loss of electron with $\varepsilon \approx 1.22$ MeV. The barometric formula $P(z) = P_0\exp(-z/z_0)$, where $P_0 =$ 1 atm, $z = H_0$, and $z_0 = 7.1$ km, was used for the altitude dependence of pressure.

Table 1. The altitude of the RE trajectory bending, the time of attachment, the charge carried by REs, and the number of REs ($H_{cl} = 18$ km, $B = 40$ μT, $\mu = 0.95$)

δ	Q_{cl}	H_0	t_{att}	S	Q_{tot}	N_e
	C	km	μs	km^2	mC	10^{16} cm^{-3}
2	75	25.5	106	19	7.3	4.6
3	144	28.4	230	37	6.5	4.1
4	210	30.5	130	54	16.9	10.5

Gamma ray emission was calculated for three moderate values of γ (from 1 to 4) at altitude H_0. Relationship (3) connects H_0 and δ. Table 1 presents the H_0 values corresponding to each of selected δ. Knowing δ and H_0, we can perform calculations. To calculate the characteristics of γ ray emission, it is formally not necessary to know the charge distribution in a cloud, the charge value Q_{cl}, and H_{cl}. However, Table 1 presents corresponding $Q_{cl} \leq 210$ C so that a reader could gain an impression of thundercloud charges maintaining selected δ.

Fishman *et al.* [1994] cited the horizontal extent (~200 km) of one of the thundercloud formations related the γ ray bursts recorded. It is improbable that charges were distributed relatively regularly over hundreds of kilometers. The electrical structure of the formations most probably represented the system of localized thunderstorm cells. The average charge densities $\rho \leq 10$ C/km^3 were measured in the thundercloud bodies with a characteristic vertical extent of $\Delta h_{cl} \approx 1$ km [Uman, 1987; Krehbiel, 1986]. If we use a disk with a thickness Δh_{cl} to simulate a charged volume, we should take from Table 1 the Q_{cl} values corresponding to a disk radius of $R_{cl} \leq \sqrt{Q_{cl}/(\pi\rho\Delta h_{cl})} \leq 2.5$ km, which is substantially less than the distances to a cloud ($H_0 - H_{cl}$) for H_0 values presented in Table 1. This circumstance allowed us to ignore a distributed charge and to replace it by a point charge when computing the upper δ boundary.

3. SIMPLIFYING ASSUMPTIONS FOR THE CASE OF THE HORIZONTAL GEOMAGNETIC FIELD

An above analysis allowed us to make the following simplifying assumptions.

(i) RREA propagates in the cross fields with the magnetic induction (B) perpendicular to the electric field strength (E). A charge develops upward to an altitude of ~$H_0(\delta, B)$.

(ii) A bending of the electron trajectories results in an abrupt narrowing of the RE flux cross section and in a rapid field relaxation so that the region with $E \approx 0$, whose width (several kilometers) has an order of the local RREA scale (l_e), appears in the vicinity of H_0.

(iii) The ratio of the Larmor electron radius for the geomagnetic field with $B \approx 40$ μT to the electron kinetic energy (ε) higher than 1 MeV is defined by the inequality $r_L/\varepsilon \approx (1 + mc^2/\varepsilon)/eBc < 130$ m/MeV. Therefore, we can assume that the energy carried by REs dissipates in the above region with $E \approx 0$. Subsequently, we assume that a point source of γ ray emission is located at altitude H_0. The radiation contribution from below is ignored since the RE flux exponentially decreases with decreasing altitude, and photons ascending from low altitudes are more intensely absorbed in the atmosphere.

(iv) Since RREA is enhanced mainly at the final stage of its development (see the beginnings of trajectories *2* and *3* in figure), it is reasonable to assume that the local RE flux (Φ_e) depends on the multiplication on the last length l_e immediately before a bending of the trajectory. Consequently, the duration of isolated γ ray bursts registered in [Fishman *et al.*, 1994] ($\Delta t_\gamma \approx$ 1–3 ms) should be equal to the time of electric field relaxation as a result of polarization of the plasma generated by REs below H_0 (see region I in figure), where the RE trajectories have not yet been strongly deflected and the field is attenuated slower than in region II in immediate proximity to H_0 (see figure). Since the layer I thickness (one length l_e) is much smaller than $H_0 - H_{cl}$, the overvoltage δ is close here to its value at altitude H_0.

(v) In a first approximation the source of γ ray emission is considered to be isotropic, and photons of all energies propagate and are absorbed in the air along the line of sight between the source and the satellite. Taking into account the bremsstrahlung indicatrix, we can substantiate this assumption by the fact that electron trajectories bend in the process of their deceleration. In the vicinity of $H_0 \sim 30$ km, the extrapolated electron path is equal to 220 and 3128 m for ε = 1 and 10 MeV, respectively. Since corresponding r_L are equal to 125 and 830 m, electrons with such energies complete 0.28 and 0.6 fractions of one rotation, respectively. When the trajectory of electrons bends, they emit in all directions, and high-energy photons are emitted mainly forward. According to [Hubbel, 1969], the characteristic length of attenuation of photons with energies of 0.05–1.0 MeV in the air at sea level varies from 41 to 121 m. The air optical thickness between H_0 and the satellite altitude corresponds to that of the 100-m layer with a density at sea level. Therefore, the photons of the above energies emitted upward were subject to only 2.5–0.8 acts of scattering on their path to the orbital station. Photons emitted in other directions, especially toward the lower half of the full solid angle, were subject to a more intense scattering. However, the assumption of an isotropic source decreases the flux of high-energy photons toward a detector since the above boundaries of photon attenuation at sea level should be multiplied by 68 in the vicinity of H_0.

4. CHARGE CARRIED BY RES

To calculate γ ray emission, it is necessary to know the number of high-energy REs (N_e) coming in the region near H_0 or the total charge Q_{tot} carried by REs into this region. Q_{tot} is limited by the time of electric relaxation ($\tau = \varepsilon_0/\sigma$) in region I (see figure), which depends on the air conductivity $\sigma = en_e(\mu_e/P)$ due to low-energy secondary electrons produced by REs themselves. According to [Huxley and Crompton, 1974], we assumed here that $\mu_e \approx 300$ cm^2 atm V^{-1} s^{-1} for electron mobility in the air. The generation of secondary electrons is hindered by electron attachment to O_2 molecules with a characteristic time of $t_{att} = f(\delta, H_0(\delta, B)) = ((k_{diss}(\delta) + k_3(\delta N)N\alpha_O)^{-1}$, where α_O is the portion of O_2 in the air, and k_{diss} and k_3 are the rates of dissociative and three-particle attachment of electrons. The t_{att} values presented in Table 1, which were calculated using the k_{diss} and k_3 values from [Aleksandrov *et al.*, 1981], satisfy the inequality $t_{att} \ll \tau \approx \Delta t_{disch} \approx \Delta t_\gamma \approx$ 1–3 ms in the altitude range of interest (see assumption (iv)). Consequently, the local equilibrium density of secondary electrons in the vicinity of H_0 can be determined from $dn_e/dt \approx n_e/\tau \approx \dot{R}_e - n_e/t_{att} = 0$ as $n_e = \dot{R}_e t_{att}$, where the specific ionization rate is expressed in terms of the RE flux (Φ_e, m^{-2} s^{-1}), and the averaged energy loss ($F^{(1)}_{r,\min} P$, eV/m) is defined as $\dot{R}_e \approx \dfrac{\Phi_e F^{(1)}_{r,\min} P}{\Delta\varepsilon}$ (m^{-3} s^{-1}). Here $\Delta\varepsilon \approx 34$ eV is the energy "cost" of one electron–ion pair generation in the air. Since the RE flux duration is not longer than the time of field relaxation

$$\tau = \frac{\varepsilon_0}{\sigma} \approx \frac{\varepsilon_0}{en_e\mu_e/P} \approx \frac{\varepsilon_0}{e\dot{R}t_{att}(\mu_e/P)} \approx \frac{\varepsilon_0\Delta\varepsilon}{e\Phi_e t_{att} F^{(1)}_{r,\min}\mu_e}, \tag{4}$$

the total charge carried by REs is estimated as follows:

$$Q_{tot} \approx e\Phi_e\tau S = \frac{\varepsilon_0\Delta\varepsilon S}{t_{att}F^{(1)}_{r,\min}\mu_e}, \tag{5}$$

where S is the area of the charge cross section in region I near H_0 (see figure). If we assume that REs are concentrated within an overturned cone, whose vertex with aperture angle φ is located at an altitude of $H_{cl} = 18$ km, then $S \approx \pi(H_0 - H_{cl})^2\tan^2(\varphi/2)$. The simulation of RREA by the MC technique, performed in [Babich *et al.*, 1998], indicated that REs are concentrated in the narrow region of angles relative to the electric force direction, whose cosines are not less than $\mu \approx 0.95$, which makes it possible to estimate S (see Table 1). The

Q_{tot} and $N_e = Q_{tot}/e$ values, calculated for three local values of δ, are presented in Table 1.

It is interesting that Q_{tot} depends only on the fundamental quantities of the air and S. Since time τ is inversely proportional to Φ_e according to (4), then Q_{tot} is independent of τ, i.e., of the discharge duration (Δt_{disch}) in the vicinity of H_0. This makes it possible to avoid a direct usage of time τ, the value of which is rather uncertain. Q_{tot} depends on δ and (indirectly) on the magnetic induction B as $t_{att} = f(\delta, H_0(\delta, B))$.

Q_{tot} was calculated neglecting ion conductivity. This is justified by the fact that the mobility of ions $\mu(O_2^-) \approx$ 2–3 cm^2 atm V^{-1} s^{-1} is by two orders of magnitude lower than that of electrons (see [McDaniel, 1964]). At the same time, the equilibrium density of ions $(\tau/t_{att})n_e$ due to the attachment of low-energy electrons of density n_e generated by RREA exceeds n_e by a factor of not more than $\tau/t_{att} \approx \Delta t_\gamma/t_{att} \le 3$ ms/0.106 ms ≈ 30 in the considered case, namely at $\Delta t_\gamma \approx$ 1–3 ms [Fishman *et al.*, 1994] and $t_{att} \le 0.106$ ms (see Table 1).

5. CALCULATION OF γ RAY EMISSION IN THE CASE OF THE HORIZONTAL GEOMAGNETIC FIELD

The motion of electrons deviated by the geomagnetic field can be on average described as the process of energy loss in the air along the x axis normal to the vertical electric field. Let x be the electron distribution function with respect to kinetic energy ε at point $x = 0$ so that the number of electrons in the $d\varepsilon$ range of energies in the vicinity of ε is $dN_e(\varepsilon) = f_e(\varepsilon)d\varepsilon$. These electrons generate the following spectrum of bremsstrahlung photons:

$$d\frac{dN_\gamma(h\nu, H_0)}{d(h\nu)} = dN_e(\varepsilon)N(H_0)\int_0^{x_\nu}\frac{d\sigma(\varepsilon', h\nu)}{d(h\nu)}dx. \quad (6)$$

Here $h\nu$ is the photon energy, $d\sigma(\varepsilon', h\nu)/d(h\nu)$ is the differential cross section of bremsstrahlung for electron with energy ε', $N(H_0)$ is the density of atomic particles at altitude H_0. At point x_ν, the energy of electrons with the initial energy ε at point $x = 0$ decreases to $h\nu$. In the region $x > x_\nu$, electrons cannot emit photons with energy $h\nu$.

The energy balance equation $d\varepsilon'/dx = -F_r(\varepsilon')$ makes it possible to replace integration over x by integration with respect to ε'. The photon spectrum at altitude H_0 (the spectrum of a source) is obtained by integrating over the $\Delta\varepsilon_{av}$ range of energies occupied by electrons at point $x = 0$

$$\frac{dN_\gamma(h\nu, H_0)}{d(h\nu)} = N(H_0)\int_{\Delta\varepsilon_{av}} f(\varepsilon)d\varepsilon\int_{h\nu}^{\varepsilon}\frac{d\sigma(\varepsilon', h\nu)}{d(h\nu)}\frac{d\varepsilon'}{F_r(\varepsilon')}. \quad (7)$$

The spectrum at the point of observation $R \gg H_0$ is calculated as follows:

$$\frac{dN_\gamma(h\nu, R)}{d(h\nu)} = \frac{dN_\gamma(h\nu, H_0)}{d(h\nu)}\frac{S_{det}}{4\pi R^2} \times \exp\left\{-\int_{H_0}^{H_0+R}\mu(h\nu)\rho(\xi)d\xi\right\} \quad (8)$$
$$\approx \frac{dN_\gamma(h\nu, H_0)}{d(h\nu)}\frac{S_{det}}{4\pi R^2}\exp[-\mu(h\nu)l_{opt}],$$

where $\mu(h\nu)$ is the mass coefficient of γ ray attenuation in the air [Hubbel, 1969]; $l_{opt} \approx \rho_1\int_{H_0}^{R+H_0}dz\exp(-z/z_0) \approx z_0\rho_1\exp(-H_0/z_0)$ is the optical density of the air layer between H_0 and $H_0 + R \approx R$; $\rho(\xi)$ and ρ_1 are the local density and the density at sea level, respectively; S_{det} is the area of a detector located at altitude $H_0 + R$. The detector area is assumed to be oriented normally to the line of sight drawn from a point source.

The spectra (7) and (8) can be used to calculate the number of photons (N_γ) and the total radiation energy (J_γ) of a source (at $z = H_0$) and at an altitude of observation ($z = R + H_0 \approx R$) in the selected range of photon energies $\Delta h\nu = h\nu_2 - h\nu_1$

$$N_\gamma(z) = \int_{\Delta h\nu}\frac{dN_\gamma(h\nu, H_0)\varphi}{dh\nu}dh\nu, \quad (9)$$

$$J_\gamma(z) = \int_{\Delta h\nu}h\nu\frac{dN_\gamma(h\nu, H_0)\varphi}{dh\nu}dh\nu, \quad (10)$$

where

$$\varphi = \begin{cases} 1 \text{ for } z = H_0 \\ \dfrac{S_{det}}{4\pi R^2}\exp[-\mu_1(h\nu)l_{opt}] \text{ for } z = R. \end{cases} \quad (11)$$

For $F_r(\varepsilon)$, it is sufficient to use the Bethe formula [Bethe and Ashkin, 1953]; for $d\sigma/d(h\nu)$, the Bethe–Heitler bremsstrahlung cross section integrated over angles for an unshielded nucleous (e.g., see [Bethe and Ashkin, 1953; Akhiezer and Berestetskii, 1959]. The density N is excluded since $Nd\sigma/d(h\nu)$ and $F_r(\varepsilon)$ are included as the ratio, and the γ ray spectrum of a source (7) is reduced to the integral

$$F_2(w, \Delta T_{av}, H_0) = \int_{\Delta T_{av}} f_{av}(T, H_0)F_1(T, w)dT, \quad (12)$$

where

$$F_1(T, w) = \frac{\alpha Z}{2\pi w}\int_{w+1}^{T+1}\frac{S_1(\gamma_1, w)}{S_2(\gamma_1)}d\gamma_1, \quad (13)$$

$$\begin{aligned} S_1(\gamma_1 w) = {} & \frac{p_2}{p_1}\left\{\frac{4}{3} - 2\frac{\gamma_1\gamma_2}{p_1p_2}\left(\frac{p_1}{p_2}+\frac{p_2}{p_1}\right)\right. \\ & + \left(l_1\frac{\gamma_2}{p_1^2} + l_2\frac{\gamma_1}{p_2^3} - \frac{l_1l_2}{p_1p_2}\right) \\ & + l_3\left[\frac{8}{3}\frac{\gamma_1\gamma_2}{p_1p_2} + \frac{w^2}{p_1p_2}\left(\left(\frac{\gamma_1\gamma_2}{p_1p_2}\right)^2 + 1\right)\right. \\ & \left.\left. + \frac{w^2}{2p_1p_2}\left(l_1\frac{\gamma_1\gamma_2}{p_1^3} - l_2\frac{\gamma_1\gamma_2 - p_2^2}{p_2^3} + \frac{2w\gamma_1\gamma_2}{p_1^2p_2^2}\right)\right]\right\}, \end{aligned} \quad (14)$$

$l_{1,2} = 2\ln(\gamma_{1,2} + p_{1,2})$, $l_3 = 2\ln[(\gamma_1\gamma_2 + p_1p_2 - 1)/w]$; $w = h\nu/mc^2$ and $T = \varepsilon/mc^2$ are the photon energy and electron kinetic energy, respectively, expressed in terms of the electron rest energy; $\gamma = 1/\sqrt{1-\beta^2}$, $\beta_{1,2} = v_{1,2}/c$; v_1 and v_2 are the electron velocities before and after the emission of a photon, respectively; $\alpha = 1/137$ is the fine structure constant; z is the atomic nucleous charge (for the air), $z = 14.5$, $\gamma_2 = \gamma_1 - w$, $p_{1,2}^2 = \gamma_{1,2}^2 - 1$; and

$$\begin{aligned} S_2(\gamma_1) = {} & \frac{\gamma_1^2}{\gamma_1^2 - 1}\left\{\ln 0.5\left(\frac{mc^2}{I}\right)^2 + \ln(\gamma_1 - 1)(\gamma_1^2 - 1)\right. \\ & \left. - \left(\frac{2}{\gamma_1} - \frac{1}{\gamma_1^2}\right)\ln 2 + \frac{1}{\gamma_1^2} + \frac{1}{8}\left(1 - \frac{1}{\gamma_1}\right)^2\right\}. \end{aligned} \quad (15)$$

The number of photons (N_γ) emitted into the $[w_1, w_2]$ range of energies and the total energy of γ ray emission (J_γ), expressed in terms of mc^2, can be calculated as

$$N_\gamma(z) = \int_{w_1}^{w_2}\varphi(w)F_2(w)dw, \quad (16)$$

$$J_\gamma(z) = \int_{w_1}^{w_2}w\varphi(w)F_2(w)dw. \quad (17)$$

The steady-state energy distribution of electrons, obtained by Babich *et al.* [1998] when simulating RREA within the scope of the ELISA program, can be approximated by the function

$$\begin{aligned} f_e(\varepsilon) = {} & k(Q_{tot}/e) \\ & \times \exp[(-\varepsilon - 2)/(\langle\varepsilon\rangle - \varepsilon_0)]/(\langle\varepsilon\rangle - \varepsilon_0), \end{aligned} \quad (18)$$

which is normalized to the total number of electrons Q_{tot}/e in the region of electron energies (ε_0) higher than 1 MeV using constant $k = \exp(0.3)$. According to result of the simulation, the average electron energy ($\langle\varepsilon\rangle$) slightly varies from 11 to 10 MeV at $\delta = 2$–8. Since we consider δ values not more than 4, $\langle\varepsilon\rangle$ was taken equal to 11 MeV. The electron energy distribution is limited from above by an energy of $\varepsilon_{max} = 51$ MeV.

Table 2 lists the values of the photon number and total energy emitted by a source located at altitude H_0; $N_\gamma(H_0)$, $J_\gamma(H_0)$, and the values of these quantities at observational point R at the satellite altitude; and $N_\gamma(R)$ and $J_\gamma(R)$ calculated for δ (altitudes H_0) presented in Table 1. Table 2 also gives the average photon energy $\langle h\nu\rangle(R) = J_\gamma(R)/N_\gamma(R)$ and the rate of photon generation $\dot{N}_\gamma(R) \approx N_\gamma(R)/\tau$ at the observational point. According to [Fishman *et al.*, 1994], it is assumed here that $\tau = \Delta t_\gamma \approx 1$–3 ms. Results obtained by Fishman *et al.* [1994] are also presented here for comparison. The integral values $N_\gamma(R)$ and $\dot{N}_\gamma(R)$, calculated for $\delta = 2$ ($Q_{cl} = 75$ C) and $\delta = 3$ ($Q_{cl} = 144$ C), are in a rather good agreement with results of measurements presented in [Fishman *et al.*, 1994]. Since $\delta = 4$ is the maximum of the considered overvoltage corresponding to the maximal charge $Q_{cl} = 210$ C, the agreement between results of measurements and calculations for smaller δ indicates that the model itself and the accepted values of the main quantities are reliable since smaller Q_{cl} and, consequently, lower δ values are more often encountered in the nature.

The average photon energy $\langle h\nu\rangle(R)$ at altitude R calculated for $\delta = 2$ and 3 is a factor of 2–2.3 higher than the value (1 MeV) obtained by Fishman *et al.* [1994] from the hardness ratios (HRs) determined as the ratios of counts in four energy channels listed in Table 2, where $h\nu_1$ and $h\nu_2$ are the channel boundaries. The calculated $N_\gamma(R)$ and $J_\gamma(R)$ distributions over the energy channels presented in Table 2 make it possible to determine HR. Fishman *et al.* [1994] stated that HRs of channel *3* to channel *2* (HR 3/2) for these events of a terrestrial origin approximately twice exceed the measured average value and are a factor of 1.4 larger than the value of the subset of γ ray flashes, especially with a hard spectrum. The HR 4/1 values indicate that the difference between both types of the phenomena is even greater. Table 2 indicates that the calculated HR 3/2 and HR 4/1 values are 3.7 and 118, respectively, for $\delta = 2$ and 2.5 and 40, respectively, for $\delta = 3$. It is necessary to compare these values with the average value and with the subset of γ ray bursts, especially with a hard spectrum [Fishman *et al.*, 1994]. On the whole, the γ ray spectrum, calculated using the electron energy distributions obtained by the MC technique, agrees with results of observations [Babich *et al.*, 2001].

The maximal energy of REs ($\varepsilon_{max} = 51$ MeV in Table 2), obtained during the simulation using the MC

Table 2. The γ ray emission in four energy channels [Fishman *et al.*, 1994] calculated for $B = 40$ μT, $\mu = 0.95$, $R = 500$ km, $S_{det} = 2 \times 10^3$ cm^2, $\varepsilon_0 = 1$ MeV, and $\varepsilon_{max} = 51$ MeV

Channel no.	$h\nu_1$	$h\nu_2$	$N_\gamma(H_0)$	$J_\gamma(H_0)$	$N_\gamma(R)$	$J_\gamma(R)$	$\langle h\nu\rangle(R)$	$\dot{N}_\gamma(R)$
	keV	keV	10^{14}	J		mc^2	keV	1/(0.1 ms)
$\delta = 2, H_0 = 25.5$ km								
1	20	50	114	58	2	0.1	26	
2	50	100	76	86	7	1	73	
3	100	300	104	290	26	10	197	
4	300	ε_{max}	160	4407	235	1334	2901	
Total	20	ε_{max}	454	4841	270	1345	2546	~9–27
$\delta = 3, H_0 = 28.4$ km								
1	20	50	102	52	8.2	0.7	44	
2	50	100	68	77	27	4	76	
3	100	300	92	258	68	25	188	
4	300	ε_{max}	143	3927	328	1593	2482	
Total	20	ε_{max}	405	4314	431.2	1622.7	1924	~34–43
$\delta = 4, H_0 = 30.5$ km								
1	20	50	261	133	56	4	37	
2	50	100	174	197	137	20	75	
3	100	300	237	661	300	108	184	
4	300	ε_{max}	366	10056	1073	4772	2273	
Total	20	ε_{max}	1037	11047	1566	4904	1600	~52–157
[Fishman *et al.*, 1994] experiment					~50–800			~15–30

technique, is in good agreement with the Q_{cl}, H_0, and H_{cl} values in Table 1 since the corresponding voltage between H_{cl} and H_0 is 100 MV. However, because of the finite potential drop between a cloud top and the altitude of trajectory bending, the RE energy distribution can be limited by a slightly lower energy. The number of photons and their average energy $\langle h\nu\rangle(R)$ at an altitude of the satellite orbit, calculated for $\varepsilon_{max} = 51$ MeV, are compared in Table 3 with the values of these quantities for a considerably lower energy $\varepsilon_{max} = 20$ MeV. It is evident that the lower ε_{max} yields a better agreement with a photon energy of 1 MeV cited in [Fishman *et al.*, 1994]. In this case $N_\gamma(R)$ and $\dot{N}_\gamma(R)$ calculated for the higher overvoltage ($\delta = 4$) also fall in the measured intervals of these quantities.

Very steep fronts of detected γ ray bursts with a rise time of about tens of milliseconds can be explained by a strong dependence of the RE flux on the electric field since the avalanche development rate strongly depends on δ, as a result of which the flux rise time is less than the time of field enhancement. The agreement between the detected γ ray spectrum [Fishman *et al.*, 1994] and the electron bremsstrahlung spectrum with $\varepsilon \sim 1$ MeV is most probably explained by the fact that photons with higher energies, emitted into a narrow solid angle, mainly propagated near the electron trajectory plane [Bethe and Ashkin, 1953; Akhiezer and Berestetskii, 1959]. These photons were less frequently scattered; the assumption of an isotropic source is not satisfied for them; and, consequently, they were not detected.

Table 3. The comparison of the γ ray burst characteristics, calculated for two values of the upper boundary (ε_{max}) of the RE energy distribution, with observations presented in [Fishman *et al.*, 1994]

δ	H_0	$N_\gamma(R)$	$J_\gamma(R)$	$\langle h\nu\rangle(R)$	$\dot{N}_\gamma(R)$
	km		mc^2	keV	1/(0.1 ms)
$\varepsilon_{max} = 51$ MeV					
2	25.5	270	1345	2546	~9–27
3	28.4	431	1623	1924	~34–43
4	30.5	1566	4904	1600	~52–157
$\varepsilon_{max} = 20$ MeV					
2	25.5	103	307	1520	~3–10
3	28.4	179	406	1160	~6–18
4	30.5	682	1292	968	~23–63
[Fishman *et al.*, 1994] experiment		~50–800			~15–30

6. VERTICAL GEOMAGNETIC FIELD (MODERATE LATITUDES)

It is interesting to consider (at least on the level of estimations) a more complicated case of UAD γ ray emission at moderate latitudes, where the vertical component of the geomagnetic field exceeds the horizontal component so that a discharge slightly bends from the vertical, i.e., from the electric force direction, and can penetrate to high altitudes. The approximations, which were made for the equatorial latitudes and considerably simplified the UAD γ ray emission calculations, are not applicable for moderate latitudes. However, we can make simple estimations. At altitudes above 30 km, an avalanche enhancement almost terminates because of an exponential decrease in the air density, and the flux of electrons ascending to high altitudes can be considered constant. Assume that the electron spectrum also remains unchanged so that the γ ray intensity depends only on air density. In particular, the collision frequency of electrons with molecules and the absorption of photons produced during radiative collisions depend only on air density (or altitude, z); i.e., $J_\gamma(z) \propto P(z)\exp(-l_{opt}(z)/\lambda)$, where $l_{opt}(z) \approx \int_z^R P(z)\,dz \approx z_0P(z)$ because $R \gg z$ and $\lambda = 1/\mu(h\nu)$. The J_γ value becomes maximal at altitude $z_{max} = z_0\ln(z_0/\lambda)$, from which it follows that $z_{max} = 35$ and 30 km for $h\nu = 100$ keV ($\lambda = 50$ m) and $h\nu = 1000$ keV ($\lambda = 100$ m) according to data from [D'Angelo, 1987]. The value $z_{max} = 30$ km is very close to the maximal altitude (H_0) calculated for the equatorial zone, especially for $\delta = 4$ (see Table 1), and to the altitude of a γ ray source estimated by Fishman *et al.* [1994]. This is related to the fact that the generation and absorption of γ quanta follow the above dependence of J_γ on z.

A source emission energy can be estimated assuming that the main portion of emission is generated at altitude z_{max}. Owing to the character of the RE energy loss and to the fact that the photon generation effectiveness follows the air density, the characteristic extent of the emitting region has an order of z_0. The RE energy input to this region is $W = Q_{tot}F^{(1)}_{r,\min}P(z_{max})z_0$. For $z_{max} = 30$ km and $Q_{tot} \approx 10$ mC, close to the values presented in Table 1, we obtain that $W \approx 10$ kJ, which corresponds to an emission energy of $J_\gamma \approx Z\frac{\langle\varepsilon\rangle(\text{MeV})}{800}W \sim 600$ J (according to the Fermi formula [Bethe and Ashkin, 1953; Akhiezer and Berestetskii, 1959]) and to the number of recorder photons $N_\gamma(R) \sim 400$ calculated on the assumption of an isotropic source of photons and propagation along the straight line.

7. CONCLUSIONS

Based on results obtained by simulating RREA for thunderstorm electric fields [Symbalisty *et al.*, 1997; Babich *et al.*, 1998, 2001, 2001a, 2001b], we have calculated the γ ray emission of UADs, taking into account the bending effect of the geomagnetic field at low latitudes and the characteristic features of the orbital experiment [Fishman *et al.*, 1994]. Using the new time and space scales of RREA presented in [Babich *et al.*, 2001, 2001a, 2001b], which are much larger than the scales used in [Roussel-Dupré *et al.*, 1996] forced us to adopt that RREA starts at lower altitudes than it was accepted in [Roussel-Dupré *et al.*, 1996]. RREA should be sufficiently increased to generate a detectable optical emission. Consequently, a dark region should exist between a point of initiation and the lower part of the observed region of emission. That is why the lower altitudes of initiation are also more acceptable. An analysis of the γ ray emission has been performed without a direct usage of the RREA scales, which are implicitly taken into account through the assumption that RREA starts immediately from a thundercloud top to become sufficiently enhanced. The number of REs was calculated based on the fact that the UAD stage responsible for γ ray generation is locally restricted to the electric field relaxation due to the electron trajectory bending by the geomagnetic field and to the polarization of the secondary plasma produced by REs themselves. Since data on the UAD geometry during the analyzed orbital experiment are absent in [Fishman *et al.*, 1994], we have used the electron angular distribution, obtained using the simulation by the MC technique, to estimate the area (S) of the RE flux transverse section. Results of an analysis are, however, not very sensitive to S since the number of REs and, consequently, the γ ray flux only linearly depend on S.

The calculated γ ray spectrum proved to be slightly harder than in [Fishman *et al.*, 1994], which is related to the accepted approximations. In particular, the maximal energy $\varepsilon_{max} = 51$ MeV in the stationary electron energy distribution (see Table 2), taken from the modeling by the MC technique, corresponds to the infinite. The factual RE distribution can be limited from above by the lower energy, owing to the finite potential drop between the cloud top and the altitude of bending. On the other hand, the average photon energy (1 MeV) cited in [Fishman *et al.*, 1994] is only the estimate, which can differ from the actual value. On the whole, the difference between the calculated and observed γ ray spectra is not significant. The number of photons calculated for the altitude of the satellite, which passed over thunderstorm formations during the observations presented in [Fishman *et al.*, 1994], is in good agreement with the number of photons actually registered in [Fishman *et al.*, 1994]. The agreement argues in favor of the UAD model proposed in [Gurevich *et al.*, 1992; Roussel-Dupré *et al.*, 1994, 1996], within the scope of which the terrestrial γ ray bursts were analyzed using the increased rate of RREA development [Roussel-Dupré *et al.*, 1996]. The consistency of the present calculations and in situ observations described in [Fishman *et al.*, 1994] has been reached because we (a) have taken into account the Lorentz force and (b) have considered UAD as the process including many avalanche generations maintaining a rather intense RE flux.

In the case of the vertical propagation of a discharge capable of penetrating to high altitudes, which is actual for high latitudes, the scales of RREA enhancement affect the final results since the avalanche multiplication terminates at high altitudes.

We should note that the duration of UAD, developing in the regime of RREA generation between the cloud top (H_{cl}) and the altitude of bending (H_0), should be much longer than the duration of the UAD stage in the vicinity of H_0, which was taken close to the observed duration of the γ ray bursts Δt_γ (see relationship (1)). The total UAD duration is limited by the time of the field shielding by plasma at lower altitudes. Consequently, REs produced in the domain between H_{cl} and H_0 carry a much larger charge than the Q_{tot} value cited in Table 1. However, the γ ray emission attenuation in the denser atmospheric layers considerably decreases the contribution of REs from the regions located much lower than H_0.

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REFERENCES

1. N. L. Aleksandrov, F. I. Vysikaĭlo, R. Sh. Islamov, *et al.*, Teplofiz. Vys. Temp. **19** (3), 485 (1981).
2. A. I. Akhiezer and V. B. Berestetskiĭ, *Quantum Electrodynamics* (Fizmatgiz, Moscow, 1959; Wiley, New York, 1965).
3. L. P. Babich, E. N. Donskoy, A. Yu. Kudryavtsev, *et al.*, Tr. Ross. Federal. Yadern. Tsentr—Vses. Nauchno-Issled. Inst. Exp. Fiz., No. 1, 432 (2001a).
4. L. P. Babich, R. I. Il'kaev, I. M. Kutsyk, *et al.*, Dokl. Ross. Akad. Nauk **379**, 606–608 (2001b).
5. L. P. Babich, R. I. Il'kaev, I. M. Kutsyk, *et al.*, Dokl. Ross. Akad. Nauk **381**, 247 (2001c).
6. L. P. Babich, E. N. Donskoy, I. M. Kutsyk, and A. Yu. Kudryavtsev, IEEE Trans. Plasma Sci. **29**, 430, (2001).
7. L. P. Babich, I. M. Kutsyk, E. N. Donskoy, and A. Yu. Kudryavtsev, Phys. Lett. A **245**, 460 (1998).
8. K. Berger, Bull. Schweiz. Eelektrotech. **69**, 353 (1978).
9. Bethe, H.A. and Ashkin, J., in *Experimental Nuclear Physics*, edited by E. Segre (Wiley, New York, 1953; Inostrannaya Literatura, Moscow, 1955].
10. M. Brook, M. Nakano, P. Krehbeil, and T. Takeuti, J. Geophys. Res. **87**, 1207 (1982).
11. N. D'Angelo, Ann. Geophys. **5**, 119 (1987).
12. G. J. Fishman, P. N. Bhat, R. Mallozzi, *et al.*, Science **264**, 1313 (1994).
13. A. V. Gurevich, G. M. Milikh, and R. A. Roussel-Dupré, Phys. Lett. A **165**, 463 (1992).
14. A. V. Gurevich, J. A. Valdivia, G. M. Milikh, and K. Papadopoulos, Radio Sci. **31**, 1541 (1996).
15. J. H. Hubbell, *Photon Cross Sections, Attenuation Coefficients and Energy Absorption Coefficients from 10 KeV to 100 GeV, NSRDS-NBS, 29*, (US Government Printing Office, Washington, DC, 20402, 1969).
16. L. G. H. Huxley and R. W. Crompton, *The Diffusion and Drift of Electrons in Gases* (Wiley, New York, 1974; Mir, Moscow, 1977)
17. P. R. Krehbiel, in *Studies in Geophysics. The Earth's Electrical Environment* (National Acad. Press, Washington DC, 1986).
18. N. G. Lehtinen, T. F. Bell, V. P. Pasko, and U. S. Inan, Geophys. Res. Lett. **24**, 2639 (1997).
19. R. A. Roussel-Dupré and A. V. Gurevich, J. Geophys. Res. **101**, 2297 (1996).
20. R. A. Roussel-Dupré, A. V. Gurevich, T. Tunnel, and G. M. Milikh, Phys. Rev. E **49**, 2257 (1994).
21. E. Symbalisty, R. Roussel-Dupré, L. P. Babich, *et al.*, EOS Trans. AGU **78**, 4760 (1997).
22. E. M. D. Symbalisty, R. Roussel-Dupré, and V. Yukhimuk, IEEE Trans. Plasma Sci. **26**, 1575 (1998).
23. M. A. Uman, *The Lightning Discharge* (Academic, San Diego, 1987).